Code No: R21016 **R10** SET - 1

II B. Tech I Semester Supplementary Examinations, September - 2014
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
All Questions carry Equal Marks
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1. a) Prove that $\int_{-1}^{1} p_m(x).p_n(x)dx = 0 \ \ if \ \ m \neq n$.

   b) Prove that $(1-x^2)p_n^1(x) = n[p_{n-1}(x) - xp_n(x)]$

2. a) Show that when n is a positive integer

   i) $J_{-n}(x) = (-1)^n J_n(x)$ ii) $J_n(-x) = (-1)^n J_n(x)$ for +ve or –ve integers

   b) Prove that $J_3(x) = \frac{8-x^2}{x^2} J_1(x) - \frac{4}{x} J_0(x)$

3. a) Show that the function given by $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is continuous at the origin.

   b) Find the analytic function whose imaginary part is $f(x,y) = x^3y - xy^3 + xy + x + y$ where z=x+iy.

4. a) Evaluate $\int_C \frac{z+1}{z^2+2z+4} dz$, where $C : |z+1+i| = 2,$ using Cauchy's integral formula

   b) Evaluate $\int_{-1+i}^{2+i} (x^2 + y^2 - ixy) dz$ along $y = x^2$

5. a) State and prove Laurent's theorem

   b) Obtain all the Laurent expansions of the function $\frac{7z-2}{(z+1)z(z-2)}$ about $z = -2$

6. a) Determine the poles and the corresponding residues of $f(z) = \frac{2z+1}{z^2-z-2}$

   b) Evaluate $\int_C \frac{e^z}{(z^2+\pi^2)^2} dz$ where $C : |z| = 4$.
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7. a) Evaluate by contour integration $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$.

   b) Evaluate $\int_0^{\infty} \frac{dx}{\left(x^2+a^2\right)^2}$.

8. a) Show that the transformation $\omega = \frac{1}{z}$ maps a circle to a circle (or) to a straight line if the former goes through the origin.

   b) Find the image of $1<|z|<2$ under the transformation $w = 2iz + 1$

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1. a) Prove that $\int_{-1}^{1} x^m p_n(x)dx = 0 \;\; if \;\; m < n$ using Rodrigue's formula.

   b) Prove that $(2n+1)p_n(x) = p^1_{n+1}(x) - p^1_{n-1}(x)$

2. a) Prove that

   i) $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{\sin(x)}{x} - \cos(x)\right]$

   ii) $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3-x^2}{x^2}\sin(x) - \frac{3}{x}\cos(x)\right]$

   b) Prove that $\int J_3(x)dx = C - J_2(x) - \frac{2}{x}J_1(x)$

3. a) If $W = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$, determine the function ϕ.

   b) If $f(z) = u + iv$ is an analytic function of z and $u - v = e^x(\cos(x) - \sin(y))$ find f(z) in terms of z.

4. a) Determine $\int_C \frac{z+4}{z^2+2z+5}dz$ where C is the circle.

   $i).|z|=1$ $ii).\ |z+1+i|=2$ $iii).\ |z+1+i|=2$

   b) Evaluate $\int_C \frac{z^2-z+1}{z-1}dz$ where C is the circle

   $i).\ |z|=2$ $ii).\ |z|=\frac{1}{2}$

5. a) Find the Taylor's expansion for the function f(z) where $f(z) = \frac{1}{(1+z)^2}$ with center at z = -i.

   b) Expand $f(z) = \frac{1}{z^3 - z - 6}$ about $i).\ z = -1$ $ii).\ z = 1$
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6. a) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle |z|=3/2 using residue theorem.

b) Evaluate $\int_C \frac{e^z}{z^2+1} dz$ over the circular path |z|=2.

7. a) Show that by the method of contour integration,
$\int_0^\infty \frac{\cos(mx)}{(a^2+x^2)^2} dx = \frac{\pi}{4a^3}(1-ma)e^{-ma}, \; a>0, \; b>0$

b) Determine the poles and residues at each pole of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$

8. a) Find the image of a circle |z-2i|=2 under the transformation w=1/z.
b) Find the image of an infinite strip 0<y<1/2 under the transformation w=1/z.

Code No: R21016 **R10** SET - 3

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1. a) Prove that $\int_{-1}^{1}[p_n(x)]^2\,dx = \frac{2}{2n+1}$

   b) State and prove Rodrigue's formula.

2. a) Prove that $\frac{d}{dx}\left(xJ_nJ_{n+1}\right) = x\left(J_n^2 - J_{n+1}^2\right)$

   b) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$

3. a) Show that the function $f(z) = \begin{cases} \frac{x^3y^3(y-ix)}{x^6+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not differentiable at the origin.

   b) If $u + iv = \frac{2\sin(2x)}{e^{2y}+e^{-2y}-2\cos(2x)}$ and $f(z) = u + iv$ is an analytic function of z, find f(z) in terms of z.

4. a) Evaluate $\int_C \left(x^2 + ixy\right)dz$ from A(1,1) to B(2,8) along

   i) the straight line AB ii) The curve x=t, y=t^3

   b) Evaluate $\int_0^{1+i}\left(x^2 - iy\right)dz$ along the paths $i).\ \ y = x \quad ii).\ \ y = x^2$

5. a) Find the Laurent's series of $\frac{7z-2}{(z+1)z(z-2)}$ in the annulus $1 < |z+1| < 3$

   b) Find the Taylor's expansion of $f(z) = \frac{2z^3+1}{z^2+z}$ about the point $z = i$.

6. a) Find the poles and residues at each pole of $\tanh(z)$.

   b) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$ where C is the circle |z|=3/2 using residue theorem
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7. a) Prove by contour integration $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx = \pi \log 2$

   b) Evaluate $\int_C \frac{e^{2z}+z^2}{(z-1)^5} dz$ where $C: |z| = 2$ using Cauchy's Residue theorem.

8. a) Find the image of the triangle with vertices at i, 1+i, 1-i in the z-plane under the transformation w=3z+4-2i.

   b) Find the image of the rectangle $-\pi < x < \pi, 1/2 < y < 1$ under the transformation w=sin(z).

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1. a) Using generating functions of Legendre's polynomial, prove that $nP_n(x) = xP_n^1(x) - P_{n-1}^1(x)$.

   b) Express x^3 using Legendre's polynomials. $P_1(x)$ and $P_3(x)$.

2. a) Show that $i)\ \cos(x) = J_0 - 2J_2 + 2J_4 \qquad ii)\ \sin(x) = 2J_1 - 2J_3 + 2J_5$

   b) Prove that $\frac{d}{dx}\left[J_n^2 + J_{n+1}^2\right] = \frac{2}{x}\left[2J_n^2 - (n+1)J_{n+1}^2\right]2$ or

   $J_n J_n' + J_{n+1}J_{n+1}' = \frac{1}{x}\left[n.J_n^2 - (n+1)J_{n+1}^2\right]$

3. a) Find the analytic function whose imaginary part is $f(x,y) = x^3y - xy^3 + xy + x + y$ where $z = x + iy$

   b) Prove that $\left[\frac{d^2}{dx^2} + \frac{d^2}{dy^2}\right] |\operatorname{Re}al\ f(z)|^2 = 2|f'(z)|^2$ where $w = f(z)$ is analytic

4. a) Find the analytic function whose imaginary part is $e^x\left(x\sin(x) + y\cos(y)\right)$

   b) Find the Principal value of $\left[\frac{\sqrt{3}}{2} + \frac{i}{\sqrt{2}}\right]^{1+i\sqrt{3}}$.

5. a) Evaluate $\int_C (x+y)dx + x^2y\,dy$ from (0,0) to (3,9) $i).\ along\ \ y = x^2 \quad ii).\ along\ \ y = 3x$

   b) If $F(a) = \int_C \frac{3z^2 + 7z + 1}{z - a}dz$ where C is the circle |z|=2. Find the value of F(1), F(3), $F^1(1-i)$ and $F^{11}(1-i)$
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6. a) Obtain the Laurent's expansion of the function $\frac{e^z}{(z-1)^2}$ in the neighborhood of its singular points and hence find its residue.

   b) Evaluate $\int_C \frac{\coth(z)}{z-i}dz$ where C is $|z|=2$

7. a) Show that $\int_0^{2\pi} \frac{d\theta}{a+b\sin(\theta)} = \frac{2\pi}{\sqrt{a^2-b^2}},\ a>b>0$ using residue theorem

   b) Evaluate by contour integration $\int_0^{\infty} \frac{dx}{(1-x^2)}$

8. a) Find the image of the infinite strip bounded by x=0 and x=π/4 under the transformation w=cos(z)

   b) Find the image of infinite strips 1/4<y<1/2 under the transformation w=1/z. Show the regions graphically.